Calculating Composite VSWR

by: M G Salak, KC9Q

Questions always arise regarding guaranteed VSWR of an antenna system, and it appears that little if any information existed regarding this subject. To begin with: Just what is VSWR? VSWR is an acronym for **V**otlage **S**tanding **W**ave **R**atio which is a ratio of maximum voltage on a transmission line to the minimum voltage. Mathematically this is expressed as:

$$VSWR = (1+|\Gamma|)/(1-|\Gamma|)$$

where $|\Gamma|$, (Reflection Coefficient) = the numerical ratio of the reflected voltage to the incident voltage.

To be more detailed:

In telecommunications and transmission line theory, the reflection coefficient is the ratio of the complex amplitude of the reflected wave to that of the incident wave. The voltage and current at any point along a transmission line can always be resolved into forward and reflected traveling waves given a specified reference impedance Z_0. The reference impedance used is typically the characteristic impedance of a transmission line that's involved, but one can speak of reflection coefficient without any actual transmission line being present.

The typical formula used when calculating transmitted reflected power for a two-way radio system is:

$$VSWR = \frac{1+SQRT[(\text{Reflected Power})/(\text{Forward Power})]}{1-SQRT[(\text{Reflected Power})/(\text{Forward Power})]}$$

Thus, if an antenna system was measured with 100 Watts forward power and 10 Watts reflected, the VSWR is calculated as:

$$VSWR = \frac{1+ SQRT(10/100)}{1-SQRT(10/100)} = \frac{1+SQRT(0.1)}{1-(SQRT(0.1)}$$

$$= \frac{1+0.3162278}{1-0.3162278} = \frac{1.3162278}{0.6837722} = \mathbf{0.9249506}$$

Going further the factor called the Reflection Coefficient, ($|\Gamma|$), is found with the following equation:

$$|\Gamma| = \frac{VSWR-1}{VSWR+1} = \frac{1.925 - 1}{1.925 + 1} = \frac{0.925}{2.925} = \mathbf{0.3162393}$$

With the Reflection Coefficient known, another factor called Return Loss (RL), can be calculated with the following formula:

RL = $-20\log_{10}(|\Gamma|) = 2\log_{10}(0.3162393$ = **9.99964,** (rounded up to **10dB**)

Return Loss (RL) is the decibel power ratio of the incident power to the reflected power.

Return Loss , (RL), is the decibel power ratio of the incident power to the reflected power. To put Return Loss (RL) into perspective the above example of 10dB has a power ratio of 10. If the forward power is 100 Watts, then taking Forward Power divided by the Power Ratio (100/10) equals the reflected power, or 10 Watts. Now using another example, a system was measured and showed: 100 Watts Forward Power and the VSWR was measures at 1.222:1, what is the reflected power?

Using the formula for Reflection Coefficient:

$$|\Gamma| = (VSWR - 1)/(VSWR + 1) = (1.222 - 1)/(1.222 + 1) = (0.222 / 2.222) = \mathbf{0.09991}$$

Now knowing the Reflection Coefficient, Return Loss can be calculated as:

$$RL = -20\log_{10}(|\Gamma|) = -20\log_{10}(0.0991) = 20.007821 \cong \mathbf{20dB}$$

The Power Ratio for 20dB = 100, thus using (Forward Power/Power Ratio), (100/10), the reflected power is 1 Watt. This above example helps to show a very important principle: As the VSWR goes down the Return Loss figure goes up.

VSWR	R.L.
1.925	10dB
1.222	20dB

A good VSWR figure for a two-way radio system is 1.1:1, and that has a Return Loss value of 26.5dB. How can you calculate the VSWR for an antenna system? The first step is to detail the components of the antenna system and define the VSWR figure for each. Typically most transmit antenna systems have an antenna, jumper cable, main transmission line, another jumper cable; possibly a transmitter combiner, or cavities and an isolator. The VSWR figure in the catalog shows the highest value that will still provide satisfactory performance for their product. However, these figures should be much lower in actual practice. Listing the typical components for an 800 Mc. System the worst case VSWR figures as shown in the catalogs should be something like this:

Item	**Manufacture**	**Model Number**	**Description**	**VSWR Limit**
Antenna	Celwave	PD1110	9.0 dBd Omni	1.50:1
Jumper	Andrew	43557-2	6’ ½” LDF, N-M	1.08:1*
Main Line	Andrew	LDF5P-50A	250’ 7/8” LDF	1.33:1*
Jumper	Andrew	43557-2	6’ ½” LDF, N-M	1.08:1*
TX Combiner	Celwave	TJD800-10S	10 Chan. Combiner	1.25:1

*VSWR is a function of the cable connectors more than the cable

What is the combined VSWR of all these parts as a system? Do you just add them up together and take the average? Actually, to come up with the combined VSWR of all these parts a figure called Composite VSWR must be calculated. Composite VSWR can be calculated with one of two formulas:

$|\Gamma|_T = SQRT[(\Gamma_1)^2 + (\Gamma_2)^2 + (\Gamma_3)^2 + \ldots\ldots.(\Gamma_n)^2]$ (Formula 1)

$VSWR_{CS}\ (1 + |\Gamma_T|) / (1 - |\Gamma_T|)$ (Formula 2)

Using the values of the components above the Composite VSWR can be calculated with the following formula:

$|\Gamma|_T = SQRT[(\Gamma_1)^2 + (\Gamma_2)^2 + (\Gamma_3)^2 + (\Gamma_4)^2 + (\Gamma_5)^2]$

where: Γ_1 = (1.50 -1) / (1.50 + 1) = 0.50 / 2.5 = 0.2000
Γ_2 = (1.08 -1) / (1.08 + 1) = 0.08 / 2.08 = 0.0385
Γ_3 = (1.33 - 1) / (1.33 + 1) = 0.33 / 2.33 = 0.1416
Γ_4 = (1.08 -1) / (1.08 + 1) = 0.08 / 2.08 = 0.0385
Γ_5 = (1.25 - 1) / (1.25 + 1) = 0.25 / 2.25 = 0.1111

Therefore: $|\Gamma|_T = SQRT(0.2000^2 + 0.0385^2 + 0.1416^2 + 0.0385^2 + 0.1111^2)$

= SQRT(0.0400 + 0.0015 + 0.2001 + 0.0015 + 0.0123)

= SQRT(0.0754)

= 0.2745

$VSWR_{CS} = (1 + |\Gamma|_T) / (1 - |\Gamma|_T|)$

= (1 + 0.2745) / (1 – 0.2745)

= (1.2745 / 0.7254)

= 1.7568

= 1.7568:1 ≅ **1.76:1**

The value of 1.7568:1 is the value that would be seen if the measurement was made at the output of the transmitter. If a 150 Watt transmitter was connected to this antenna system what would the reflected power be?

$|\Gamma|$ = (1.7568 - 1) / (1.7568 + 1) = 0.7568 / 2.7568 = 0.27542

$RL = -20\log_{10}(|\Gamma|) = -20\log_{10}(0.27542)$ = 11.2284 dB

The power ratio for 11.2284 dB = $10^{(11.2284/10)} = 10^{(1.12284)}$ = 13.269055

Reflected Power = (Forward Power / Power Ratio) = (150/13.269055) = 11.30454 Watts.

This VSWR value is far out-of-line (hopefully), from a typical installation. If the figures were replaced with more realistic values the VSWR figures should be something like:

Item	Manufacture	Model Number	Description	VSWR Limit
Antenna	Celwave	PD1110	9.0 dBd Omni	1.09:1
Jumper	Andrew	43557-2	6’, ½” LDF, N-M	1.08:1*
Main Line	Andrew	LDF5P-50A	250’, 7/8” LDF	1.05:1*
Jumper	Andrew	43557-2	6’, ½” LDF, N-M	1.08:1*
TX Combiner	Celwave	TJD800-10S	10 Chan. Combiner	N/A **

*VSWR is a function of the cable connectors more than the cable.

** The combiner should always show a good match due to the Isolator Panel.

With the above note regarding the combiner in mind, the VSWR of an antenna system should be measured after the combiner network.

$$|\Gamma|_T = SQRT[(\Gamma_1)^2 + (\Gamma_2)^2 + (\Gamma_3)^2 + (\Gamma_4)^2 + (\Gamma_5)^2]$$

where: $\Gamma_1 = (1.09 -1) / (1.09 + 1) = 0.09 / 2.09 = 0.0430$

$\Gamma_2 = (1.08 -1) / (1.08 + 1) = 0.08 / 2.08 = 0.0385$

$\Gamma_3 = (1.05 - 1) / (1.05 + 1) = 0.05 / 2.05 = 0.0244$

$\Gamma_4 = (1.08 -1) / (1.08 + 1) = 0.08 / 2.08 = 0.0385$

Therefore: $|\Gamma|_T = SQRT(0.0430^2 + 0.0385^2 + 0.0244^2 + 0.0385^2)$

$= SQRT(0.00185 + 0.0015 + 0.0006 + 0.0015)$

$= SQRT(0.00545)$

$= 0.0738$

$VSWR_{CS} = (1 + |\Gamma|_T) / (1 - |\Gamma|_T|)$

$= (1 + 0.0738) / (1 – 0.0738)$

$= (1.0738 / 0.9262)$

$= 1.1594$

$=1.1594:1 \cong$ **1.16:1**

$|\Gamma| = (1.1594 - 1) / (1.1594 + 1) = 0.1594 / 2.1594 = 0.07382$

$RL = -20\log_{10}(|\Gamma|) = -20\log_{10} (0.07382) = 22.6365$ dB

The power ratio for 22.6365 dB = $10^{(22.6365/10)} = 10^{(2.28365)} = 193.3593$

Reflected Power = (Forward Power / Power Ratio) = (150/193.3593) = 0.775758 Watts.

With all the facts and figures presented here, the question still remains: What is a good guaranteed VSWR for our antenna system? **When all the figures are examined it can be concluded that adding 10% to the highest anticipated VSWR of the antenna system should be satisfactory.** The two components that normally show the highest VSWR figure is the antenna and/or the cavity filter system. Both of these devices are frequency dependent with respect to the VSWR.

An antenna has a finite bandwidth at which it will provide the published gain figure. Typically an 800 Mc. High gain fixed station antenna has a 35 Mc. Bandwidth with a VSWR specification of 1.5:1. If a trunked radio system has one or more channels working at these limit, then the antenna will probably produce a VSWR reading of 1.5:1 at these frequencies. Other frequencies may be less, but the highest system VSWR will be in the area of 1.5:1. Adding 10% to the 1.5:1 figure will yield a VSWR figure of 1.653:1 which should absorb any higher VSWR figures found elsewhere in the transmission line. The following table shows the possible transmission line VSWR figures needed to project the composite figure figure to a 1.652:1 level:

Device	**VSWR**	**Remarks**
Antenna	1.50:1	Nominal
Jumper	1.17:1	High
Main Line	1.20:1	High
Jumper	1.17:1	High
Composite VSWR	1.6527	

Figuring a modest 3 dB loss in a Transmitter Combiner, the output of a 150 Watt transmitter would be reduced to 75 Watts. Inserting 75 Watts into a mismatch of 1.653:1 will produce 4.54 Watts of reflected power.